Comment on Professor Timberlake's Squared Rule for the Equilibrium Value for the Marginal Utility of Money

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In both volume 1[1] and in volume 2[2] of *The Review of Austrian Economics*, Professor Timberlake states his "squared" rule for the equilibrium value of the marginal utility of money, to wit:

$$MU_m = \frac{MU_c}{P_c^2},$$

where:

MU_m is the marginal utility of money,

MU_c is the marginal utility of the "composite good"[3], and

P_c is the price of the composite good.[4]

This rule is based on the neoclassical theory of the consumer. Now, even if we ignore the insurmountable analytical problems concerning the concepts of the marginal utility of a composite good, and accept the analysis on its own terms (i.e., using the neoclassical analytical apparatuses of comparative statistics/equilibrium analysis), Professor Timberlake's rule is erroneous. That is, even if the concepts of a composite good and its marginal utility are accepted, his formulation remains a faulty application of neoclassical comparative statics/equilibrium analysis. Thus I shall deal with it on its own terms.

According to the neoclassical theory of the consumer, each consumer is assumed to maximize a utility function subject to a budget constraint. (Professor Timberlake simplifies matters somewhat by considering a "typical individual."[5] Thus:

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maximize: $U[X_i]$ for $i = 1, \ldots n$,

subject to: $Y_i = \Sigma P_i \cdot X_i$,

where:

$U(X_i)$ is the typical individual's utility function,

X_i is the quantity of the i-th good purchased by the typical individual,

P_i is the price of the i-th good,

Y_i is the wealth in monetary terms of the typical individual, and

n is the number of goods.

Assuming the utility function exhibits the necessary mathematical conditions in terms of continuity, differentiability, and so on, the first-order conditions for maximization are:

$$\frac{MU_i}{P_i} = L \text{ for } i = 1 \ldots n, \text{ and}$$

$$Y = \Sigma P_i \cdot X_i,$$

where:

MU_i is the typical consumer's marginal utility of the i-th good and

L is a Lagrange multiplier.

From the first-order conditions, the following can be derived:

$$\frac{MU_1}{P_1} = \frac{MU_2}{P_2} = \ldots = \frac{MU_n}{P_n},$$

or, in more useful form:

$$\frac{MU_i}{MU_j} = \frac{P_i}{P_j}, \text{ for } i = 1 \ldots n, \text{ and } i \neq j.$$

That is, the (subjective) rate at which the typical individual is willing to substitute between any two goods (given by the ratio of that individual's marginal utilities of the goods) must be equal to the (objective to him) rate at which other people are offering to exchange the same two goods (given by the ratio of the prices of the goods).

In his analysis, Professor Timberlake assumes (implicity) that the typical consumer's utility function and budget constraint may be represented as including but two goods (money and a composite good):

$$U = U\ (C,M) \text{ and}$$

$$Y = P_c \cdot C + P_m \cdot M$$

where:

C is the quantity of the composite good purchased by the typical consumer, and

M is quantity of money purchased by the typical consumer.

Thus he (implicitly) derives the maximizing conditions as:

$$\frac{MU_c}{P_c} = L$$

$$\frac{MU_m}{P_m} = L$$

$$Y = P_c \cdot C + P_m \cdot M,$$

from which he (implicitly) derives his equilibrium condition:

$$\frac{MU_c}{P_c} = \frac{MU_m}{P_m}$$

This is a legitimate neoclassical formulation provided that P_m is the nominal price of money, that is, the price of money in terms of itself, to wit: one (1). This condition is essential if the budget constraint is to be consistent. Thus, where:

Y is in terms of dollars (\$),

P_c is in terms of dollars per unit of the composite good (\$/c),

C is in units of the composite good (c), and

M is in dollars (\$),

$$\$ = \$/c \cdot c + P_m \cdot \$ \text{ or } \$ = \$ + P_m \cdot \$.$$

It is immediately obvious that for the budget constraint to be consistent, the price of money must be dimensionless, specifically $P_m = 1$. Thus, substituting one (1) for the price of money in our budget constraint yields

$$\$ = \$ + 1 \cdot \$ \text{ or } \$ = \$ + \$,$$

in which case the dimensions of the budget constraint are consistent.

Note that Professor Timberlake's use of the reciprocal of the price of the composite good $[1/P_c]$ as the price of money, that is

$$P_m = 1/P_c = 1/(\$/c) = c/\$,$$

results in the budget constraint having the following dimensions:

$$Y = P_c \cdot C + P_m \cdot M,$$
$$\$ = \$/c \cdot c + c/\$ \cdot \$ \text{ or } \$ = \$ + c,$$

which obviously are inconsistent.[6]

Unfortunately, Professor Timberlake missed this crucial point. This failure resulted in his substitution of the reciprocal of the price of the composite good for the price of money in his equilibrium condition, yielding:

$$\frac{MU_c}{P_c} = \frac{MU_m}{1/P_c} \text{ or } \frac{MU_c}{P_c^2} = MU_m;$$

or, in more useful form,

$$\frac{MU_c}{MU_m} = P_c^2.$$

In this latter form, Professor Timberlake's error and its source are clear. The rate at which money and the composite good may be exchanged is given by the money price of the composite good, not by its square. It was the improper substitution of "the" real price of money (the reciprocal of the price of the composite good) for the nominal price of money in the budget constraint that caused the squared rule.

We see then that even within the framework of neo-classical analysis, Professor Timberlake's rule is untenable. In addition, from an Austrian perspective, there is an even more fundamental problem with the neoclassicists' attempted mathematization, via the use of the infinitesimal calculus, of economic analysis. In this case, it appears in the form of the unwarranted and implicit assumptions that the composite good be infinitely divisible, that the law of diminishing marginal utility not hold with respect to subdivisions (new units?) of the last unit of the composite good, and that marginal utilities be infinitely divisible, whatever that may mean.

Consider a more appropriate formulation of the neoclassical theory. All notation is as previously given. However, the utility function and budget constraint are expressed in terms of "real variables"—that is, "real income" and "real (money) balances." Real balances is, of course, the name given to the purchasing power of actual ("nominal") money balances. Since there is only one nonmoney good (the composite good), both real income and real balances must be, and are, in terms of units of the composite good. That is, their dimensions are:

$$M/P_c = \$/(\$/c) = c \text{ and}$$
$$y = c$$

where

$$M/P_c = \text{real balances and}$$
$$y = \text{real income.}$$

Thus, the utility function and budget constraint are:

$$U = U(C, M/P_c) \text{ and}$$
$$y = M/P_c + C.$$

Note that the dimensions of the budget constraint are consistent.

The first-order conditions for a maximum are:

$$MU_C = L,$$
$$MU\ (M/P_c) = L, \text{ and}$$
$$y = M/P_c + C.$$

Thus

$$MU_C = MU\ (M/P_c).$$

That is, the marginal utility of the last "unit of real balances" purchased by the consumer must be equal to the marginal utility of the last unit of the composite good purchased by the consumer.

But, the problem is that units of real balances are multiples (or possibly fractions) of units of actual money balances. Thus, to use Professor Timberlake's example of a tripling of the money stock, assume the following:

1. The initial stock of money is \$100 ($M = \$100$),
2. The initial stock of the composite good is 500c ($C = 500c$), and
3. The initial price of the composite good is \$1/c, ($P_c = \$1/c$).

Then

$$y = \$100/(\$1/c) + 500c \text{ or } y = 600c.$$

Let the money stock triple and, in (assumed) consequence thereof, the price of the composite good do likewise.

Then:

1. Then the new stock of money is \$300,
2. The stock of the composite good is unchanged, and
3. The new price of the composite good is \$3/c.

Note that real income is unchanged:

$$y = \$300/(\$3/c) + 500c \text{ or } y = 600c.$$

Then, in the initial situation the marginal utility of the five-hundredth unit of C is equal to the marginal utility of the last unit of real balances, which is the amount of actual money necessary to buy a unit of the composite good. In this situation, the unit of real balances is \$1. That is, the marginal utility of the five-hundredth unit of C is equal to the marginal utility of the one-hundredth dollar.

In the subsequent situation, the marginal utility of the five-hundredth unit of C is still equal to the marginal utility of the last unit of real balances. However, in this case, the unit of real balances is \$3. That is, the marginal utility of the five-hundredth unit of C is equal to the marginal utility of the two-hundred-ninety-eighth through the three-hundredth dollars, *taken as a unit.* Unless the composite good is infinitely divisible, it is meaningless to speak of the marginal utility of the last unit of actual money, i.e., the three-hundredth dollar, for it will only fetch one-third of a unit of C, if such may be purchased. And, even if this is possible, the marginal utility of the three-hundredth dollar would be equal to the marginal utility of the "last third" of the five-hundredth unit of C, and not to one-third of the marginal utility of the five-hundredth unit of C. That is:

Initial situation: MU (100th \$) = MU (500th C);

Subsequent situation: MU (298th, 299th, and 300th \$s, as a unit) = MU (500th C).

However, this does not mean that

MU (300th \$) = 1/3 · MU (500th C) but, rather that

MU (300th \$) = MU (last 1/3 of 500th C).

Thus, for Professor Timberlake's analysis to be correct, these latter two equations must be identical. That is:

MU (last 1/3 of 500th C) = 1/3 · MU (500th C).

And this must hold true for any possible increase or decease in the money stock and attendant price change. But, as mentioned already, this requires that the composite good be infinitely divisible, that the law of diminishing marginal utility not hold with respect to subdivisions (new units?) of the last unit of the composite good, and that marginal utilities be infinitely divisible.

The foregoing analysis demonstrates some of the errors in the analysis leading to the squared rule. These errors are, unfortunately, typical manifestations of what is perhaps the most serious problem in monetary/business cycle/macroeconomic analysis. This problem is inappropriate aggregation, which arises out of attempts to make economic analysis mathematically tractable and to make economic data grist for the mill of statistical (i.e., econometric) analysis.

Notes

1. Richard H. Timberlake, Jr., "A Critique of Monetarist and Austrian Doctrines on the Utility and Value of Money," *The Review of Austrian Economics*, vol. 1 (1987), pp. 93–94.

2. Richard H. Timberlake, Jr., "Reply to Comment by Murray N. Rothbard," *The Review of Austrian Economics*, vol. 2 (1987), pp. 189–97.

3. Of course, except in the imaginations of some people, including many famous economists, "composite good(s)" do not exist. The attempt to create mathematically tractable models of the economy has led many to accept such fiction(s) as reasonable approximations to reality. Unfortunately, this has resulted in much confusion and misunderstanding, and in fact to the foisting of much relatively low level mathematical exercises and games as advanced theoretical economics.

4. The system of notation used in this comment is somewhat different from Timberlake's, but the reader should have no trouble reconciling them.

5. *The Review of Austrian Economics*, vol. 1, p. 93.

6. If the editors of journals required economists to assign dimensions to the variables in their mathematical and statistical equations, many errors could be avoided.