“On Gallaway and Vedder on Stabilization Policy”
William Barnett II and Walter Block
4/27/2005

These are the derivations of the aggregate, productivity-adjusted, real wage based on: 1) marginal, not average, productivity and without assuming perfect competition in input and/or output markets − the general case; and, 2) average productivity and without assuming perfect competition in input and/or output markets; and, derivations of: 3) the marginal product of labor for the CES function; and, 4) the condition for equality between: a) Gallaway and Vedder’s aggregate, productivity-adjusted, real wage based on a Cobb-Douglas production function ; and, b) based on a constant elasticity of substitution production function
for:
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(Please, no comments about the problems of aggregation, with respect to any of the variables. We are well aware of them, but this is derivation is in regard to a mainstream paper that uses the orthodox methods, from assumptions to mathematics, in all their glory.)

The correct formulation of a productivity adjusted real wage (ARW) is:
ARW = $(W/P)/MP_L = (1+1/E)/(1+1/e)$; where:
W = nominal wage rate;
P = relevant price or price level;
w = W/P = real wage rate
MP_L = marginal product of labor;
E = price elasticity of demand for output; and,
e = nominal wage price elasticity of supply of labor.
And, below:
p = profits;
Q = quantity of output;
L = quantity of labor;
R = rental price of capital goods;
K = quantity of capital goods.

Let profits be given by: p = PQ – WL – rK. Maximizing p wrt to L yields:
$\partial p/\partial L = (P + Q\partial P/\partial Q)(\partial Q/\partial L) - (W + L\partial W/\partial L) = 0 \Rightarrow (P + Q\partial P/\partial Q)(\partial Q/\partial L) = (W + L\partial W/\partial L)$
Then: $P((1+(Q/P)(\partial P/\partial Q))(\partial Q/\partial L) = W(1+(L/W)(\partial W/\partial L)) \Rightarrow P(1+1/E)MP_L = W(1+1/e) \Rightarrow (W/P)/MP_L = w/MP_L = (1+1/E)/(1+1/e)$. Q.E.D.

2. Derivation of: $w/AP_L = ((1+1/E)/(1+1/e))(\varepsilon_L)$

From 1, above: $(W/P)/MP_L = (1+1/E)/(1+1/e) \Rightarrow (W/P) = ((1+1/E)/(1+1/e))(MP_L) \Rightarrow (W/P)/AP_L = ((1+1/E)/(1+1/e))(MP_L/AP_L) = ((1+1/E)/(1+1/e))(\varepsilon_L)$ Q.E.D.

3. Derivation of: MP_L for CES function

$Q = (cK^{-\alpha} + (1-c)L^{-\alpha})^{-1/\alpha} \Rightarrow \partial Q/\partial L = -(1/\alpha)(cK^{-\alpha} + (1-c)L^{-\alpha})^{-(1/\alpha)-1}(-\alpha)(1-c)(L^{-\alpha-1}) =$
$(cK^{-\alpha} + (1-c)L^{-\alpha})^{-1/\alpha} (1-c)/(cK^{-\alpha} + (1-c)L^{-\alpha})(L^{\alpha+1}) = (Q/L)((1-c))/(cK^{-\alpha} + (1-c)L^{-\alpha})(L^{\alpha}) =$
$MP_L = AP_L((1-c)/(c(L/K)^{\alpha} + (1-c)) = AP_L/((c/(1-c))(L/K)^{\alpha} + 1)$ Q.E.D.

4. Condition for equality G&V's ARW w/CD and w/CES: $K/L = ((c-ac)/(a-ac))^{1/\alpha}$
Assuming perfect competition in all markets: $ARW = w/MP_L = 1$

CD production function: $Q = AK^aL^{1-a}$
$MP_L = (1-^a) AK^aL^{-a}$
$AP_L = AK^aL^{1-a}/L = AK^{\alpha}L^{-a}$
$MP_L = (1-a)AP_L$
$w/MP_L = 1$
$w/(1-a)AP_L = 1$
$w/AP_L = 1-a$

CES production function: $Q = (cK^{-\alpha} + (1-c)L^{-\alpha})^{-1/\alpha}$
$MP_L = (cK^{-\alpha} + (1-c)L^{-\alpha})^{-1/\alpha} (1-c)/(cK^{-\alpha} + (1-c)L^{-\alpha})(L^{\alpha+1})$
$AP_L = (cK^{-\alpha} + (1-c)L^{-\alpha})^{-1/\alpha}/L$
$MP_L = AP_L/((c/(1-c))(L/K)^{\alpha} + 1)$
$w/MP_L = 1$
$w/(AP_L/((c/(1-c))(L/K)^{\alpha} + 1)) = 1$
$w/AP_L = 1/((c/(1-c))(L/K)^{\alpha} + 1))$

Then, equality of w/AP_L between CD and CES production functions requires:
$1/((c/(1-c))(L/K)^{\alpha} + 1)) = 1-a$
$K/L = ((c-ac)/(a-ac))^{1/\alpha}$ Q.E.D.